SPECTRAL MODELING OF WAVE PROPAGATION IN COASTAL AREAS WITH A HARBOR NAVIGATION CHANNEL

by

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ABSTRACT

This study presents a comparison of numerical model results and laboratory experiments of wave propagation in a coastal area with a harbor navigation channel. The results of wave models SWASH, SWAN and HARES are compared with physical model results in order to investigate the performance of these models. It turns out that HARES, a 2D parallel spectral wave model based on the Mild-Slope Equation with non-linear damping, yields the most accurate results and a computational time that is only a small fraction of the time needed by SWASH. It appears that the large computational effort and resolution required by full 3D time-dependent wave models like SWASH may prevent them from exploiting their full potential accuracy, even though they contain all relevant physics for wave propagation. Furthermore, the phase-resolving wave modeling approach used by both HARES and SWASH yields more accurate results than the phase-averaged approach used by SWAN when channel reflection and diffraction effects are involved, which can be important in the vicinity of harbor navigation channels. HARES combines the advantages of a stationary and two-dimensional calculation (enabling sufficient model resolution at low cost) with a phase-resolving modeling approach. This underlines the ongoing applicability of mild-slope wave models like HARES in practice and makes them a preferable tool for the design of harbor layouts.

1. INTRODUCTION

The numerical wave model HARES is a stationary phase-resolving 2D model based on the Mild-Slope Equation. Since the first introduction of the modeling concept by Berkhoff (1972), the development of mild-slope-type wave models has become widespread. HARES is often used in practice for the modeling of wave penetration into harbor areas. Similar models are e.g. PHAROS (Hurdle et al., 1989), TELEMAC ARTEMIS and MIKE21 EMS. In recent years, 3D non-hydrostatic wave models like SWASH (Zijlema et al., 2011) have been introduced and increasingly used. Conceptually such full 3D models can yield more accurate results than 2D mild-slope models, because they take into account non-linear wave propagation effects like Stokes waves and cnoidal waves; yet, some downsides are the large amount of computational time needed and possible difficulties to obtain stable results in practice. Another type of wave model, often used for computing wave conditions outside a harbor, is a phase-averaged spectral wave-energy model like SWAN (Booij et al., 1999). This model is often applied in near-shore areas, but is less suitable for wave penetration in regions where diffraction and reflection play an important role.

In 2014 a comparison was made between a SWASH and SWAN wave model and 3D laboratory experiments of a harbor navigation channel area (Dusseljee et al., 2014). Recently, Svašek Hydraulics has remodeled these laboratory experiments using the mild-slope model HARES. This paper presents the HARES results and the comparison with the physical and other numerical model results.

The paper has two main objectives:

1. Comparison of HARES results with experimental results in order to investigate model accuracy;
2. Comparison of HARES results with SWAN and SWASH results to assess the performance of the mild-slope model HARES compared to wave models of quite different characters.

2. PHYSICAL MODEL

The physical model setup of a harbor navigation channel region is described in Dusseljee et al. (2014) and Riezebos (2014). This section gives a brief summary of the model setup. At prototype scale, the 3D laboratory experiment represents an existing navigation channel towards a harbor, with a 15 km

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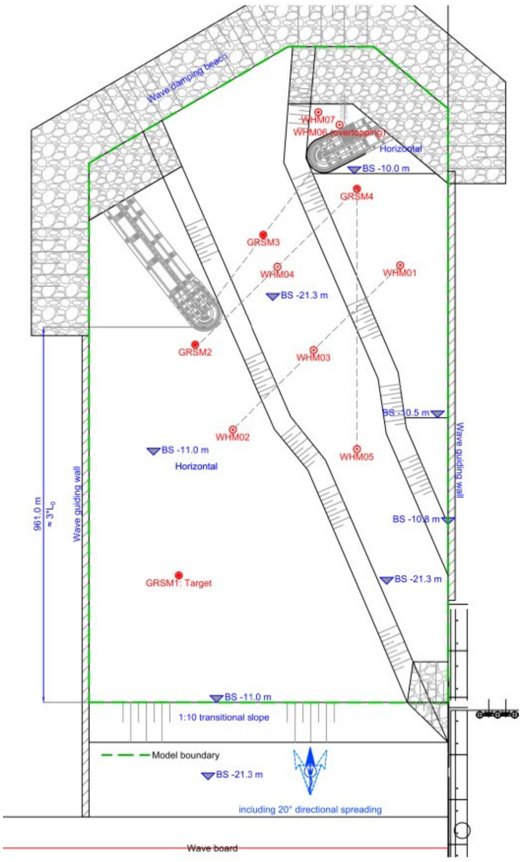
long straight access channel; see Fig. 1 for the experimental set-up of the harbor entrance. Waves are generated on deep water, at a water depth of -21.3 m, which is also the water depth of the access channel. A gentle transition slope (1:10) guides the waves towards the actual foreshore of the harbor. The side slopes of the access channel are designed with a 1:5 slope. The width of the channel at the bottom level is 250 m near the harbor entrance (in between the two breakwaters) and 170 m further offshore.



Entrance channel

Western breakwater

Figure 1: Set up of physical model in basin (Riezebos, 2014).

Physical scale-model tests were performed in a 3D wave basin of Deltares in the Netherlands, equipped with a multi-directional wave generator, able to generate short-crested random waves. The wave generators are equipped with active wave absorption in order to prevent the re-reflection of waves (generated by structures and bathymetry in the basin) upon the wave paddles.

The physical model is Froude-scaled at scale 1:60, implying that gravity-based wave processes are included correctly as compared with prototype conditions. The applied orientation of the incident waves is 0° (normal to the wave paddles) for all tests. The waves approach the access channel with an incident angle of 23°. At the basin end a wave-damping beach is installed to dissipate outgoing wave energy. A normally-distributed directional spreading (with 20° standard deviation) is applied.

Wave conditions are measured at several locations, as indicated in Fig. 1. The open circles are standard resistant-type wave gauges (WHM). The closed circles are directional wave gauges (GRSM).

Two distinguished wave scenarios are presented, with relatively small wave angles between the incident wave direction and channel axis. A single-peaked wind-sea wave spectrum (case C1) and a double-peaked spectrum including both local wind conditions and swell (case C2) are applied (see Fig. 2). In both the numerical and physical models, wave boundaries are applied as 2D wave spectra with a directional wave spreading of 20°. The water level for both conditions is CD+0.8 m.

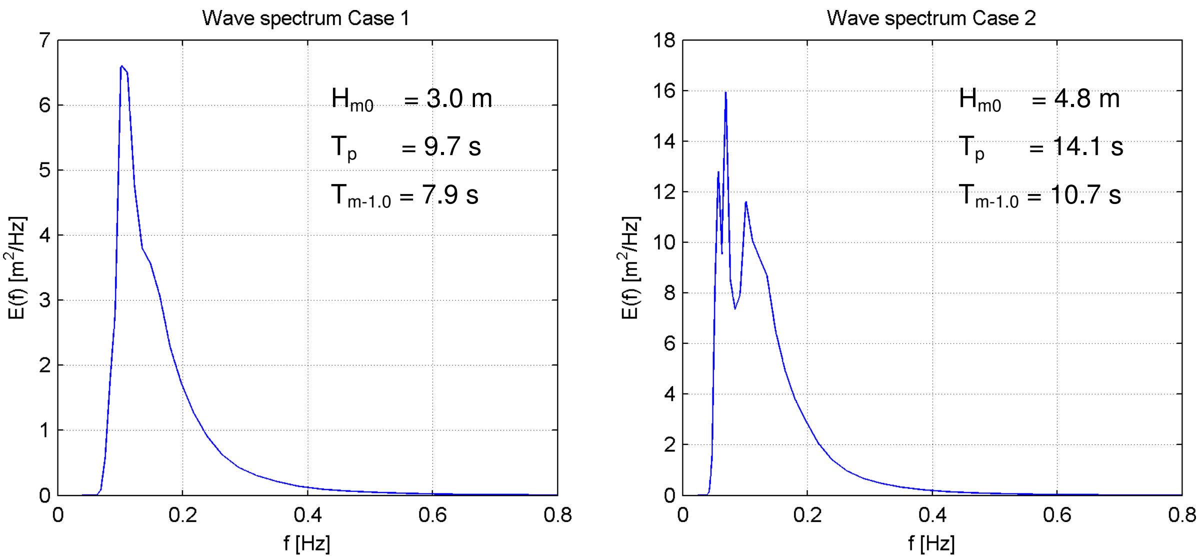


Figure 2: Imposed wave spectra for case C1 (left) and C2 (right).

3. HARES NUMERICAL MODEL

3.1 Introduction

The focus of this paper is on wave modeling using HARES; results of SWASH and SWAN model simulations have been taken from Dusseljee et al. (2014) for comparison. In this section, first a general description of the mild-slope wave model HARES is given; subsequently, the present numerical model set-up is presented. Finally, numerical model results as well as a comparison with experiments and other models will be given.

3.2 Description of the mild-slope wave model HARES

The Finite Element model HARES ("*HArbor RESonance*") has been developed in-house by Svašek Hydraulics. HARES is a one- or two-dimensional parallel model based on the Mild-Slope Equation (Berkhoff, 1972), with non-linear damping terms incorporated, which can be applied to simulate the propagation and possible resonance of waves in coastal zones and harbor areas. It is especially useful in harbor and breakwater optimization studies and for determining the natural frequencies of a harbor basin. The Mild-Slope Equation is a practical example of a Helmholtz problem, in which the complex amplitude of a harmonic wave component is computed in the model domain; within the field of Helmholtz-type problems, the Mild-Slope Equation is characterized by the relative dominance of damping/dissipation and spatial variation of bathymetry (and hence wave celerity).

Within HARES the Mild-Slope Equation has been discretized using the Continuous Galerkin approach, employing an unstructured triangular flexible mesh of linear finite elements (yielding a second-order accurate discretization in space). This enables the user to fit boundaries accurately into the model area and increase resolution in the region of interest in a flexible way, without the need for nesting of grids. The Finite Element discretization gives rise to global systems of linear equations, which are solved on a parallel cluster. HARES employs a rather efficient form of parallelism.

Model input includes incoming wave characteristics (e.g. wave height/period and wave direction), domain bathymetry, water level, reflection and transmission coefficients for hydraulic structures and other physical borders.

HARES includes the following model features:
- One- or two-dimensional wave propagation over topography;
- Diffraction around obstacles;
- Refraction and shoaling;
- Wave damping due to bottom friction and wave breaking (depth- or steepness-induced);
- Reflection at boundaries (full or partial);
- Combined (full or partial) reflection and (full or partial) transmission at internal boundaries (e.g. breakwaters);
- Uniform incident wave at seaward boundaries;

- Monochromatic versus spectral computations (frequency spreading and directional spreading);
- Consistent spectral treatment of damping terms (optional);
- Solving systems in parallel using a direct solver (MUMPS) or iterative solver (BiCGSTAB);
- Very fast solution algorithm for spectral computations thanks to efficient reuse of matrices.

The software package HARES has been developed by Svašek Hydraulics for several decades. The scientific basis of the Mild-Slope Equation as implemented in HARES has been given in many text books, e.g. Dingemans (1997); a lot of numerical pioneering work on Finite Elements within Svašek Hydraulics has been performed by Labeur, which have been reported in his PhD thesis (2009). Among the more recent improvements and extensions to the model are the full parallelization of HARES (2014-2015), the incorporation of combined reflection-transmission boundaries along breakwaters (2017) with a very user-friendly user interface, and the addition of a consistent and accurate spectral treatment of bottom friction and wave breaking based on the entire wave spectrum (2017), inspired after the spectral wave model SWAN. A speedup of the model of over a factor 10 has been reached thanks to efficient matrix reuse (2017).

3.3 Model setup

Wave conditions

The wave conditions for case C1 and C2 are discretized by dividing the 2D wave spectra into a large number of frequencies with a 0.01 Hz interval, see Fig. 3. This results in 28 frequency bins for C1 and 30 frequency bins for C2.

The red lines represent the frequencies that are modeled in HARES, including a line precisely at the peak frequency. For each frequency bin the amount of wave energy is determined. The maximum frequency in this case is set to is 0.33 Hz, which corresponds to a wave period of 3.0 seconds. This cut-off frequency results from a trade-off between two practical considerations: on the one hand a 3.0 s wave can still be resolved on the present computational grid, on the other hand more than 95% of the total spectral energy is resolved by the discretized frequencies. Moreover, all energy in the tail of the spectrum (f > 0.33 Hz) is assigned to the last frequency bin.

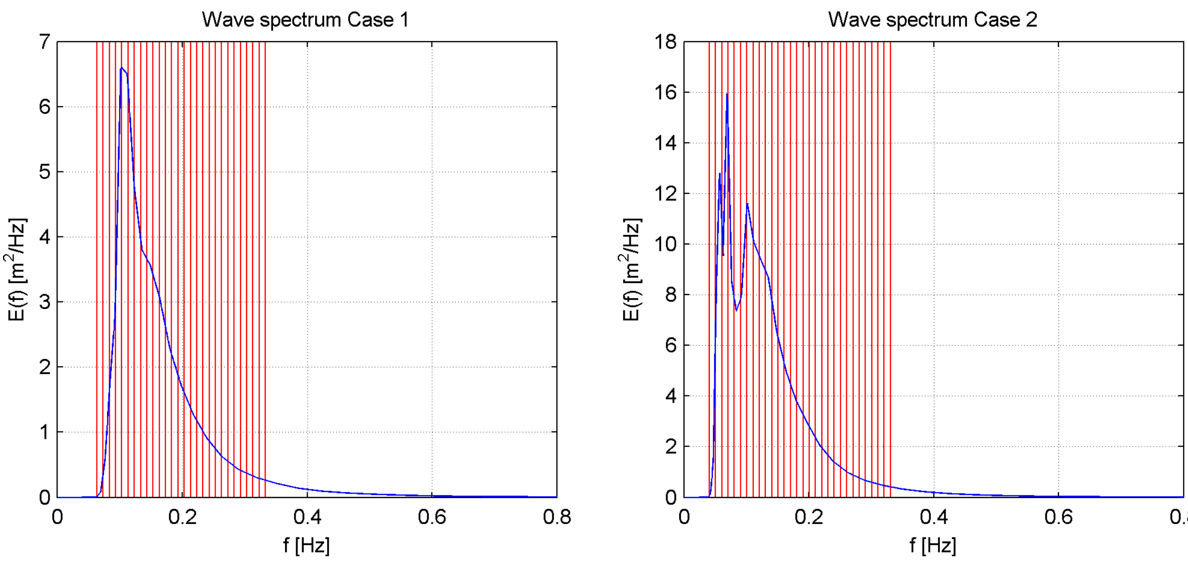


Figure 3: Discretization of 2D wave spectrum for case C1 (left) and C2 (right).

For each frequency a directional wave spreading is applied with a standard deviation of 20°. This directional spreading is schematized by distributing the wave energy over a main wave direction and wave directions up to 45° at both sides of the main wave direction, with an interval of 5°. The total wave energy is 100%. Fig. 4 depicts the distribution of the wave energy, normalized by the peak value, over the 19 discrete wave directions for a single frequency.

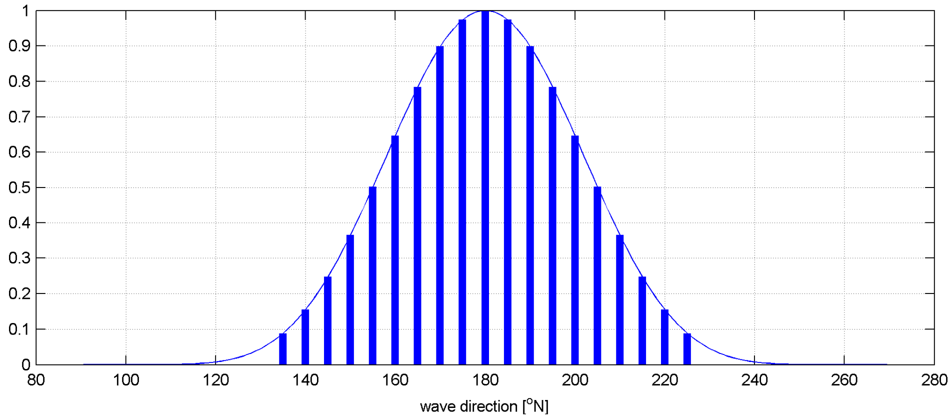


Figure 4: Directional spreading for case C1 and C2.

The above results in a total of 532 individual wave components for case C1 (28 frequency bins times 19 directions) and 570 wave components for case C2 (30 frequency bins times 19 directions). As described in Section 3.2, HARES applies this set of wave components on behalf of a consistent spectral treatment of bottom friction and wave breaking based on the entire wave spectrum, inspired after the model SWAN. Hence, the above set of wave components is considered as a single wave spectrum by HARES.

Model domain and bathymetry

The computational domain and bathymetry used within HARES are based on the corresponding SWASH input files (Dusseljee D.W., et al., 2014). For the HARES bathymetry the SWASH bathymetry and structure height are combined. The resulting HARES bathymetry is depicted in Fig. 5.

The flexible mesh of the computational domain consists of 1.06 million triangular elements with a size of approximately 1.5 meters on average. For both cases a water level of CD+0.8 m is used, in accordance with SWASH input files.

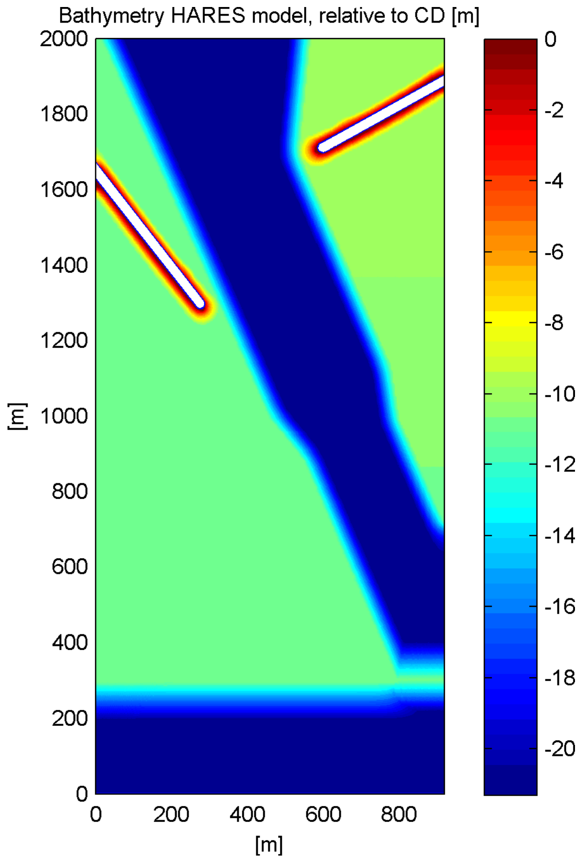


Figure 5: Model domain and applied bathymetry.

For the breakwaters a reflection coefficient of 0.53 for case C1 and 0.55 for case C2 is applied, in accordance with Riezebos (2014). The vertical basin walls have a reflection coefficient of 1.0 and the wave-damping beach has a reflection coefficient of 0.0. Besides these reflection coefficients, domain bathymetry and spectral wave components, only a few model settings are required: a wave bottom friction parameter C_f = 0.001 (suitable for the present concrete-wall laboratory set-up) and two wave breaking criteria, i.e. a maximum wave height (relative to water depth) of 0.88 and a maximum wave steepness of 0.14. Hence, the number of settings to “tune” the model are rather limited.

3.4 Results and discussion

In this section, HARES results for both scenarios are presented and compared with results by the physical model and the numerical models SWASH and SWAN.

The presence of an approach channel can change local wave conditions considerably. When travelling waves approach the channel under an angle larger than the critical angle (to the channel-normal axis), they are fully reflected at the downward channel slope (total reflection). When the wave direction angle compared to the channel-normal axis is smaller than the critical angle, waves may (partially) cross the channel. This phenomenon is described in literature, e.g. Zwamborn and Grieve (1974).

In cases C1 and C2, most of the waves in the spectrum have an approach angle beyond the critical angle, resulting in a considerable concentration of wave energy at the left side of the approach channel. Fig. 6 shows spatial distributions of the significant wave height H_{m0} as computed with SWASH, SWAN and HARES for both cases.

Table 1 summarizes (for each output location) the results (H_{m0}) of the physical model (PHM) and the SWASH, SWAN and HARES wave models for case C1. Results for case C2 are given in Table 2. The columns with SWASH, SWAN and HARES results also show the relative difference (in percentages) with the measured wave heights in the physical model.

From Fig. 6 it is notable that, generally, the HARES results contain more wave energy on both sides of the navigation channel (compared to SWASH and SWAN) as well as inside the navigation channel and the harbor basin itself. Especially the wave reflection pattern along the right side of the channel (on the seaward side of the eastern breakwater) is noteworthy. As can be seen from physical model results at the points GRSM4 and WHM01 within Table 1 and 2, this energy content crossing the navigation channel as computed by HARES is in accordance with the physical model experiments.

For case C1 the HARES model gives significantly better results than the SWASH and SWAN model. The over-all average absolute error (based on the absolute value of all error percentages) for HARES is only 3%, for SWASH and SWAN these are 24% and 18% respectively. Moreover, the SWASH and SWAN results are all (systematic) underestimations of experimental results, whereas the HARES results give both small under- and overestimations. The computational time on a 16-core cluster-computer for HARES was only 21 minutes, which is 0.7% of the SWASH computation (48 hours and 20 minutes).

Location	PHM	SWASH		SWAN		HARES	
	H_{m0} [m]	H_{m0} [m]	diff	H_{m0} [m]	diff	H_{m0} [m]	diff
GRSM1	3.03	2.51	(-17%)	2.82	(-7%)	3.04	(0%)
GRSM2	3.63	3.14	(-13%)	3.36	(-7%)	3.67	(1%)
GRSM3	2.36	1.80	(-24%)	2.06	(-13%)	2.53	(7%)
GRSM4	2.50	1.85	(-26%)	1.64	(-34%)	2.64	(6%)
WHM01	2.81	2.07	(-26%)	1.60	(-43%)	2.88	(2%)
WHM02	3.55	3.25	(-8%)	3.30	(-7%)	3.64	(3%)
WHM03	2.77	1.93	(-30%)	2.15	(-22%)	2.65	(-4%)
WHM04	2.60	1.88	(-28%)	2.11	(-19%)	2.62	(1%)
WHM05	2.73	1.85	(-32%)	2.10	(-23%)	2.61	(-4%)
WHM07	1.30	0.89	(-32%)	1.33	(2%)	1.22	(-6%)
Average absolute error			24%		18%		3%
Computational time		48 hrs 20 min		17 min		21 min	

Table 1: Results for case C1, significant wave height H_{m0} [m].

For case C2 the HARES model yields better results as well. The average absolute error for HARES is 10%, for SWASH and SWAN these are 15% and 20% respectively (with systematic underestimation). Again, the computational time needed by HARES is only a fraction of the time needed by SWASH. Computational times needed by HARES and SWAN have equal order of magnitude for both cases.

Location	PHM	SWASH		SWAN		HARES	
	H_{m0} [m]	H_{m0} [m]	diff	H_{m0} [m]	diff	H_{m0} [m]	diff
GRSM1	4.80	4.37	(-9%)	4.64	(-3%)	4.84	(1%)
GRSM2	5.33	5.26	(-1%)	5.42	(2%)	5.61	(5%)
GRSM3	3.56	2.96	(-17%)	2.93	(-18%)	3.43	(-4%)
GRSM4	4.31	3.82	(-11%)	2.37	(-45%)	3.87	(-10%)
WHM01	4.36	3.42	(-22%)	2.42	(-44%)	4.21	(-3%)
WHM02	5.36	5.26	(-2%)	5.44	(2%)	5.76	(7%)
WHM03	4.42	3.40	(-23%)	3.16	(-29%)	3.81	(-14%)
WHM04	3.98	3.14	(-21%)	3.07	(-23%)	3.56	(-11%)
WHM05	4.66	3.47	(-26%)	3.03	(-35%)	3.54	(-24%)
WHM07	2.11	1.69	(-20%)	2.07	(-2%)	1.63	(-23%)
Average absolute error			15%		20%		10%
Computational time		49 hrs 20 min		14 min		26 min	

Table 2: Results for case C2, significant wave height H_{m0} [m].

Finally a comparison of the modeled and measured wave spectra at the wave gauges is depicted in Fig. 7 (for case C1) and Fig. 8 (for case C2). These wave spectra show that for almost all locations the HARES spectrum comes closest to the measurement. The SWASH and SWAN results mostly show significantly lower energy densities compared to measurements, especially for case C1. For case C2 the differences are less pronounced, yet HARES results still come closest to the experimental data. Applying wave damping (due to bottom friction and wave breaking) on the full spectrum in HARES is an important improvement, as previous tests (not shown in this paper) with damping applied at individual wave bins only, instead at the total wave spectrum, gave less accurate results.

As pointed out previously by Dusseljee et al. (2014) the high-frequency waves as computed by SWASH (f > 0.13 Hz) show significant damping, which has been attributed to a relatively low spatial resolution of the SWASH model (i.e. a grid size of 3.0m, equal to SWAN). This results in an inaccurate reproduction of the high-frequency part of the spectrum; indeed, as can be seen from Fig. 7 and 8, wave energy is lacking in the right-hand-side part of most spectra. Sensitivity simulations performed by Dusseljee et al. (2014) indicate that a considerable improvement of SWASH results might be expected if the horizontal resolution is enhanced to 0.5 m and the vertical resolution to 3 layers (instead of the present 2 layers). Yet, such simulations were considered quite unfeasible due to constraints regarding computational time, even though the present computational effort ratio between SWAN/HARES and SWASH is about 1% already (as pointed out by Tables 1 and 2).

These considerations tend to the conclusion that the 3D time-dependent model SWASH, although it contains all relevant physics for spectral wave propagation, is generally outperformed by time-independent and spectra-based models like SWAN and HARES, which prevents it from exploiting its full potential accuracy. This implies that 2D spectral wave models form a more practical tool for use in real-life cases where relatively wide wave spectra are involved.

From Fig. 6 it can be observed as well that the significant wave height results for SWAN are more smooth than those for HARES and SWASH; this is due to the fact that SWAN is a phase-averaged wave model, whereas SWASH and HARES are intrinsically phase-resolving. From Table 1 and 2 it can be noted that (for both scenarios) SWAN performs relatively well along the wave-ward side of the channel, where the measurement locations GRSM2 and WHM02 are situated. A substantial part of the incoming multi-directional wave spectrum travels at angles where waves (theoretically) do not cross the navigation channel, which causes wave energy to pile up against the western slope of the channel, giving rise in turn to breaking waves and a “sharp edge” of wave energy. As observed by Dusseljee et al. (2014), wave heights along the lee-ward side of the channel are significantly underpredicted by SWAN compared to measurements and also to SWASH results (see locations GRSM4 and WHM01 in Table 1 and 2), which may be caused by inaccuracies regarding the effects of channel reflection and diffraction within SWAN (due to its phase-averaged modeling approach). Apparently, the use of a phase-resolving modeling approach may be advantageous when waves cross a navigation channel under a certain angle: both HARES and SWASH give better significant wave height results than SWAN along the lee-ward side of the channel (even though high-frequency waves in SWASH are known to show too much damping).

In view of the above observations, we may state that mild-slope models to a certain extent combines "the best of both worlds" conceptually. On the one hand, the 2D time-independent spectral approach which is characteristic for mild-slope models provides them with an efficiency advantage above full 3D time-dependent models, which include all relevant physics but whose resolution requirements cause them to be outperformed. On the other hand, the mild-slope approach still remains a phase-resolving approach which includes phase-related wave phenomena like diffraction accurately, which can give an advantage (for geometries like the present one) above the phase-averaged approach employed by spectral wave-energy models. Both advantages can be observed from the comparison between SWASH, SWAN and HARES as depicted in Figures 6-8 and Tables 1-2.

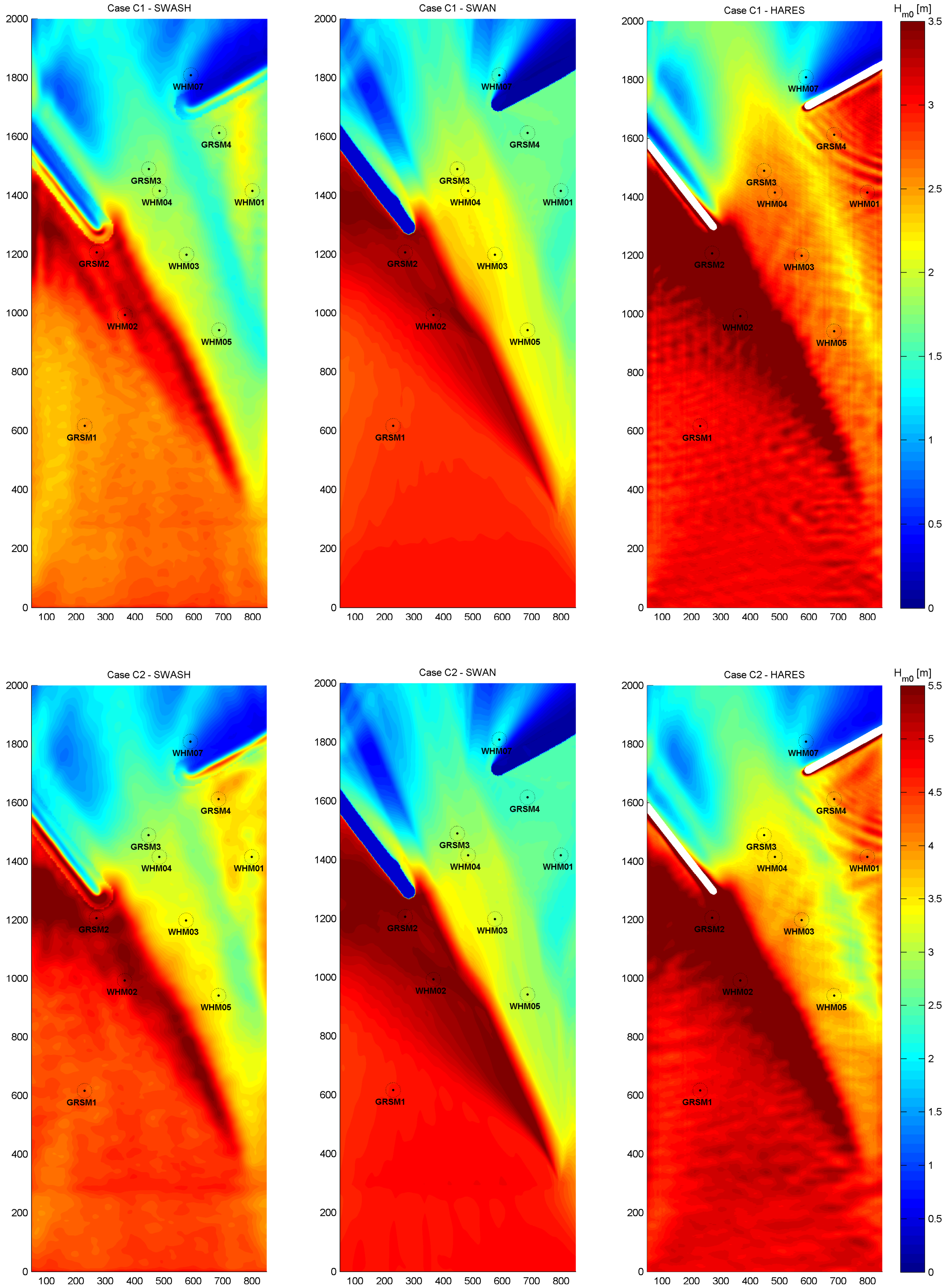


Figure 6: Spatially varying significant wave height Hm0 [m], as computed by SWASH (left), SWAN (middle) and HARES (right). Top: scenario C1. Bottom: scenario C2.

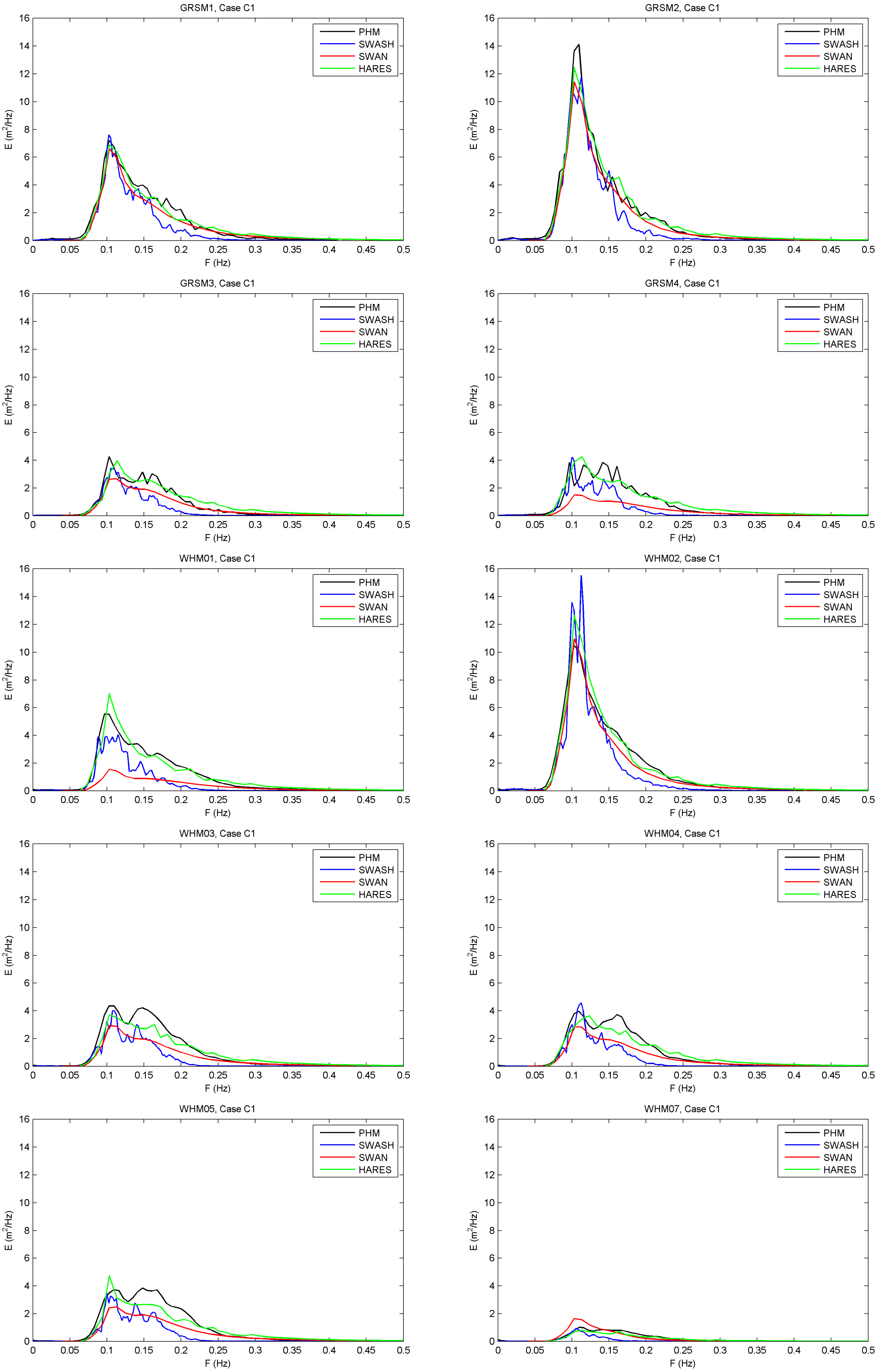


Figure 7: Frequency wave spectra at the wave gauges for all models for case C1. (Black: physical model, blue: SWASH, red: SWAN and green: HARES).

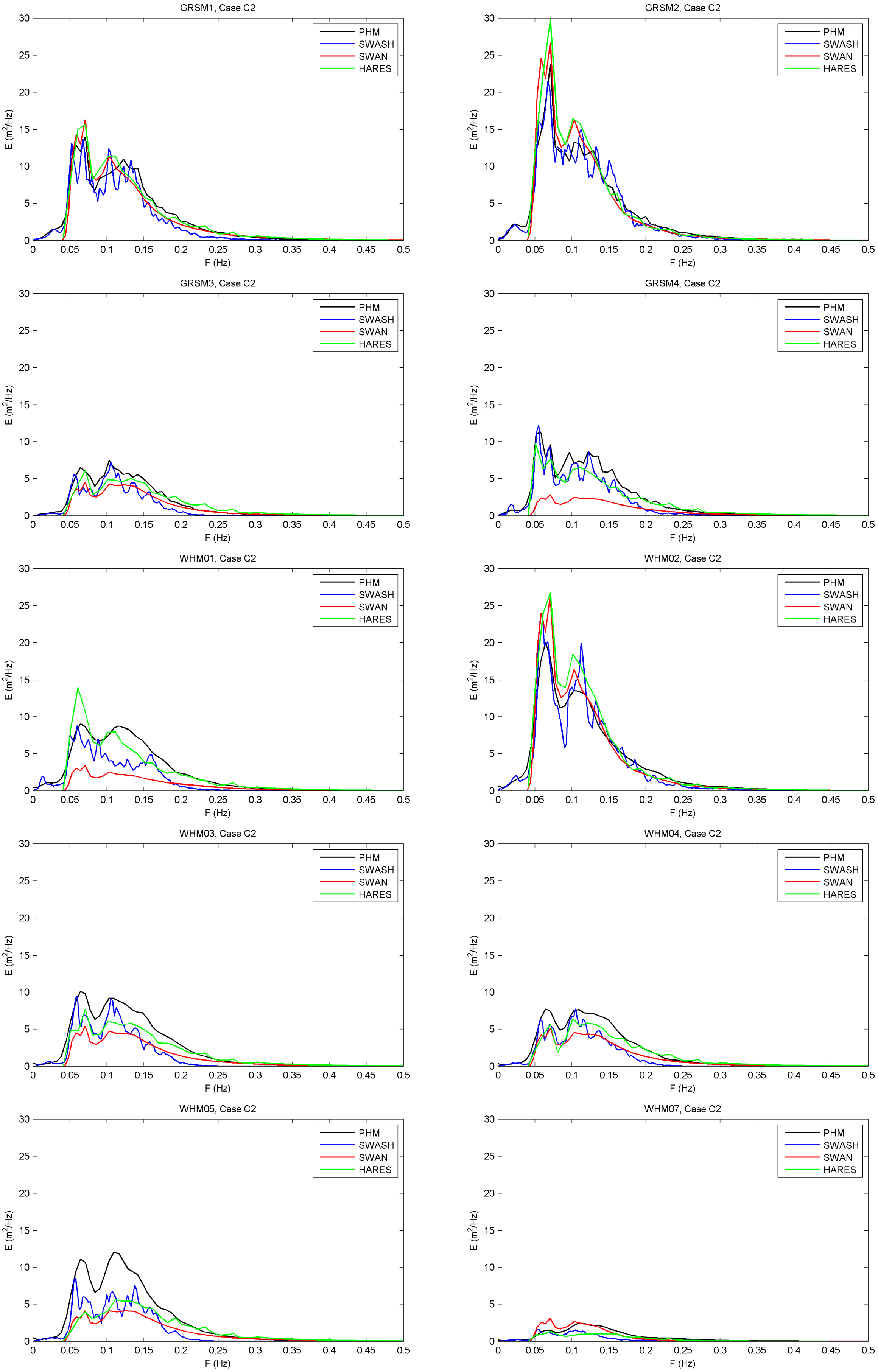


Figure 8: Frequency wave spectra at the wave gauges for all models for case C2. (Black: physical model, blue: SWASH, red: SWAN and green: HARES).

4. EXAMPLE: WAVE TRANSMISSION ALONG BREAKWATERS

The latest improvements of the HARES numerical wave model include the addition of a consistent and accurate spectral treatment of bottom friction and wave breaking, based on the entire wave spectrum, and the incorporation of combined reflection-transmission boundaries along breakwaters. The case of the approach channel discussed above showed the positive effect of spectral damping treatment. As for the combined reflection-transmission boundaries, an example will be presented in this section.

To this end a small real-life harbor is considered (using a fictitious but realistic bathymetry), see Fig 9. The harbor is protected by two breakwaters, a large one and a smaller one. In this case a very high water level is applied, resulting in a wave transmission coefficient of 50% for both breakwaters. Furthermore, realistic reflection coefficients are used for all other model boundaries.

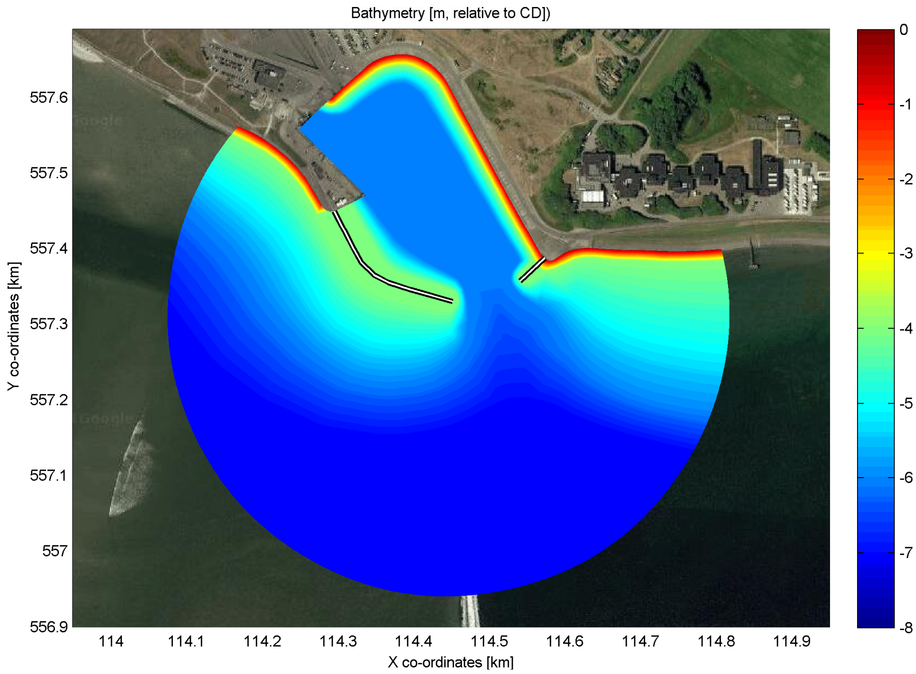


Figure 9: Model domain and applied bathymetry for transmission example.

In order to present the influence of wave transmission, two cases are modeled: a case with no transmission and 30% partial reflection of the breakwaters and a case with both transmission (50%) and partial reflection (30%) along both breakwaters. The wave condition imposed is T_p = 6.0 s, H_{m0} = 1.0 m and a wave direction of 240°N. Again, this wave condition is subdivided into a large number of frequencies and directions.

Model results are given in Fig. 10 for the case without transmission (top panel) and for the case with 50% transmission (center panel). The differences in wave height for both cases are presented in the bottom panel of Fig. 10. In a circular area inside the harbor, the average wave height is computed and plotted in the figures as well.

The wave height behind the main breakwater increases with almost 0.50 m (which is 50% of the 1.0 m offshore wave height) due to transmission; hence, the wave transmission process over and through the breakwater is modeled adequately. Some processes like diffraction and refraction, however, hinder a fully quantitative 1-to-1 comparison of the wave transmission effects.

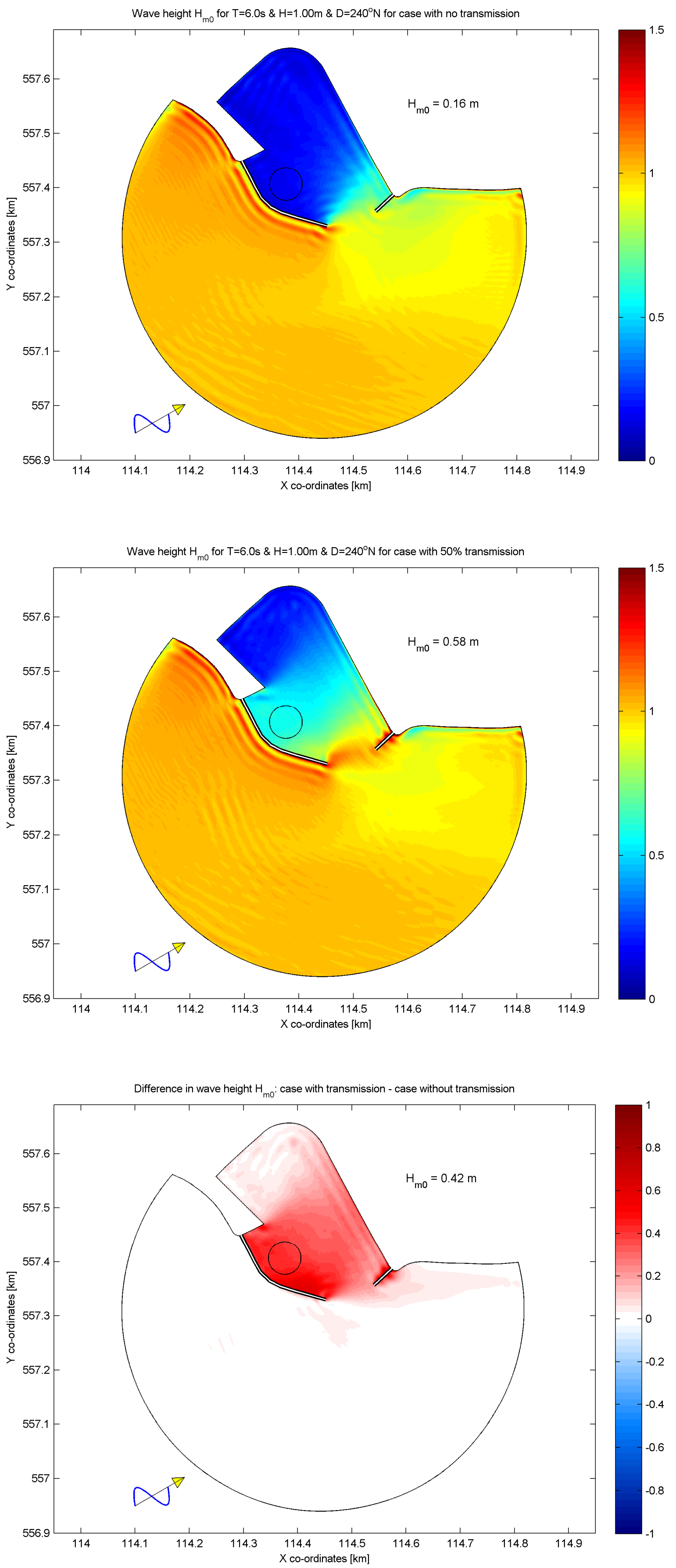


Figure 10: Spatially varying significant wave height Hm0 [m] for case without transmission (top), case with 50% transmission (center) and difference in wave height between both cases (bottom).

5. CONCLUSION

Wave propagation throughout a coastal area with a harbor entrance navigation channel has been investigated using a physical scale experiment and three numerical models of very different character. For 10 different output locations, significant wave heights and wave energy density spectra computed by the numerical mild-slope model HARES have been compared to experimental data and results by the numerical models SWASH and SWAN.

From this comparison it is concluded that, for the investigated cases, the mild-slope model HARES gives significantly more accurate results than SWASH and SWAN, without significant tuning of default model input coefficients. Furthermore, the computational time needed by HARES is less than 1% of that needed by SWASH, whereas the computational speeds of HARES and SWAN are comparable.

It appears that the strict resolution requirements by full 3D time-dependent models like SWASH (which include all relevant physics) make it virtually unfeasible to perform spectral wave computations at reasonable computational cost without causing the high-frequency waves to dampen out too fast. On the other hand, it has been shown that a phase-averaged modeling approach (as employed by spectral wave-energy models like SWAN) may cause inaccuracies regarding channel reflection and diffraction effects, which are important in the present case of a harbor with navigation channel. Because 2D spectral mild-slope models like HARES are both time-independent and phase-resolving (instead of phase-averaged), whereas the correct spectral treatment of damping terms is maintained, we may state that they combine “the best of both worlds” from a conceptual point of view.

An additional case of a harbor with breakwater crests just above the water level shows that HARES is able to model partial wave transmission over and through breakwaters in a realistic manner.

These results underline the ongoing applicability and convenience of mild-slope-type numerical wave models for harbor design purposes (or other situations where multiple variants have to be investigated and computational time is limited), despite the fact that the mild-slope modeling concept is relatively old compared to more recent scientific developments like phase-averaged spectral wave-energy models (SWAN) or fully 3D non-hydrostatic models (SWASH).

We conclude that wave models based on the Mild-Slope Equation, in combination with efficient computational procedures for a fast and accurate spectral treatment of bottom friction and wave breaking based on the entire wave spectrum, are quite competitive to numerical models that are conceptually more sophisticated. In practice, mild-slope models like HARES remain a preferable tool for the design of harbor layouts.

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